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RESEARCH ARTICLE

A Mathematical Model to Prevent the Growth of Bacterial Colony Produce Toxic Substances

Rajan Kumar Sharma[1], Pradeep Kumar Singh[1], Santosh Kumar Dixit[2], Akhilesh Gupta[1] Syed Suboor Aziz[3] and Sudhir Singh[4]

[1]Department of Basic Sciences and Humanities, Pranveer Singh Institute of Technology, Kanpur (U.P.), India.
[2]Department of Applied Mathematics, Amity school of Engineering and Technology, Amity University Patna (Bihar) , India.
[3]Department of Master of Computer Applications, Pranveer Singh Institute of Technology, Kanpur (U.P.), India.
[4]Department of Applied Science, Accurate Institute of Management and Technology, Greater Noida, (U.P), India.

*Corresponding Author
Rajan Kumar Sharma

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Abstract: Given their significance in our surroundings, the study of bacterial population models is highly relevant to population dynamics. They are crucial to the applications of various industries, including food production and fermentation technology. When dangerous bacteria enter the body and proliferate, it can lead to a bacterial infection and sickness. These infections can range in severity from localized problems like skin abscesses to widespread problems like sepsis, and they can affect different sections of the body.In the present study is oriented for controlling the growth of bacterial colonyby mathematical modelling based on Malthusian population dynamics growth modelis discussed in limited environment for which mortality depends on increasing density of bacteria's and the concentration of the toxic products produced by bacteria in the cultures.

Keywords: Population of bacteria,toxic products, mortality, environment.

INTRODUCTION

Bacteria are single-celled organisms that develop in diverse environments[1].These organisms can live on earth, water, and inside the intestines of humans as well as other living beings. The relationship between bacteria and humans is very complex. Sometimes bacteria help with our digestive system. Furthermore, bacteria are terrible because they can cause illnesses like pneumonia and methicillin-resistant Staphylococcus aureus (MRSA).Bacteria are classified in the forms of prokaryotes, which are single-cell organisms with a simple interior structure, without nucleus, with DNA which floats independently and collected protein with aminoacid**[2,3].**Bacterial cells typically have two protective layers: an inside cell membrane and an exterior cell wall. However, certain bacteria lack a cell wall, such as Mycoplasma. But some bacteria, like mycoplasma,have not any cell wall.There are three types of bacteria sizewise. First are round bacteria called Cocci, second are cylindrical type are called as Basilli; and the third are spiral bacteria, called Spiriila and fourth are vibrowhich are curved type [**4]**

Numerous dangerous bacterial species are the cause of infections [**5,6**]. They were the source of numerous infectious diseases, including dental decay, syphilis, diphtheria, pneumonia, and tuberculosis. which are unfriendly, and antimicrobial medications can counteract their effects.

One of the most crucial ways to prevent illness in all of them is to take precautions. Sterilization, heat, disinfectants, UV radiation, pasteurization, boiling, and other methods can eradicate the majority of these pathogenic microorganisms.

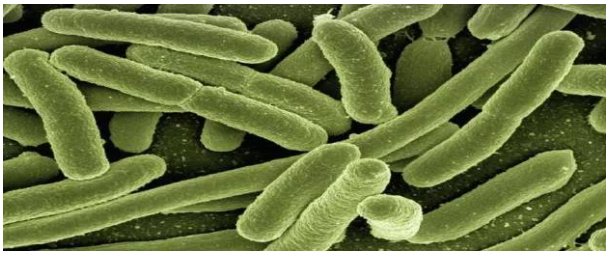

Fig1: Image of Bacteria

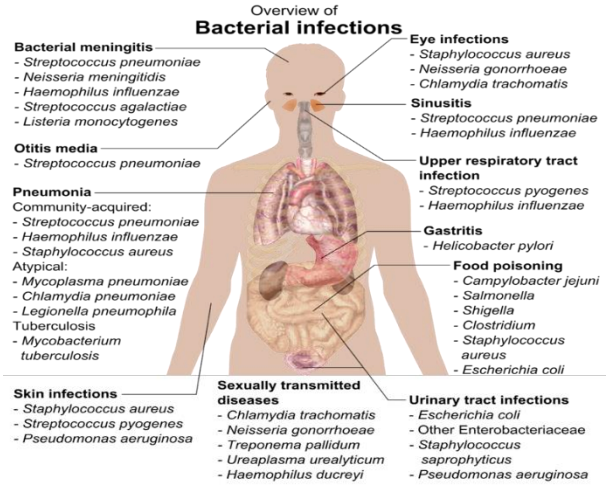

Fig 2: Bacterial infections and related bacterial species

In India basically some types of bacteria have been found named as asE.coli, S.pneumoniae, K.pneumoniae, S.aureus and A.baumanii to be the deadliest pathogens now days**[7].**A study in The Lancet states that diseases brought on by the aforementioned germs collectively killed over 6.8 lakh people in India in 2019. Although E. Coli, which causes pneumonia, urinary tract infections, and diarrhea, among other things, killed up to 1.6 lakh people nationwide, S. pneumoniae, K. pneumoniae, S.

aureus, and A. baumannii killed 1.4 lakh, 1.3 lakh, 1.2 lakh, and 1.1 lakh people, respectively.

The Lancet study is based on 33 species' deaths from bacterial illnesses, including all five of the previously listed species.In total, the report shows, 13.7 people died due to bacterial infections in India in 2019**[8, 9].**The other common bacteria responsible for infection-related deaths included Salmonella Typhi, non-typhoidal Salmonella and Pseudomonas aeruginosa among others.

Table-1: Top five death causing bacteria in India

S.No.	Pathogen	Death Count	Death rate per 100,000 population
1	E. Coli	1.6 Lakh	16.1
2	S-Pneumonia	1.4 Lakh	14.4
3	K-Pneumoniae	1.3 Lakh	12.2
4	S-Aureus	1.2 Lakh	12.8
5	A-Baumannii	1.1 Laks	11

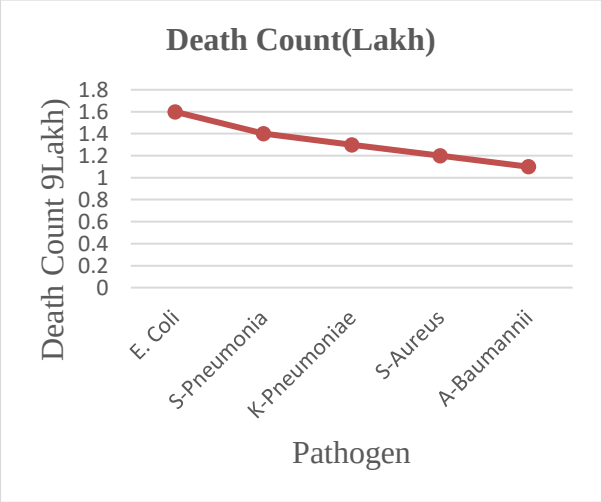

Fig-3: Mortality due to bacteria

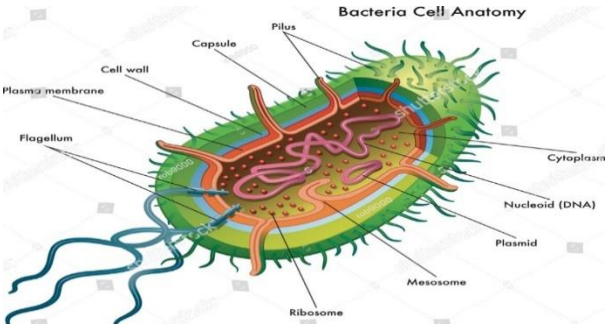

Fig-4: Bacteria Cell

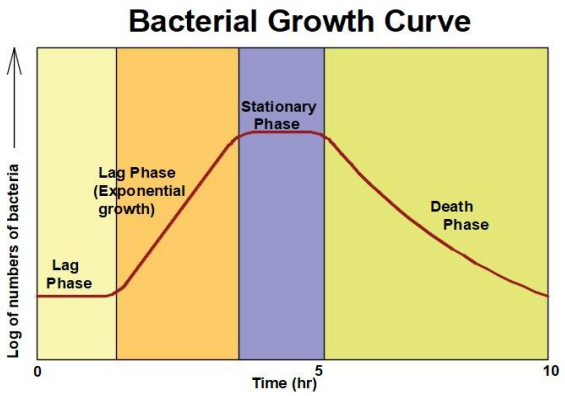

Fig 5: Bacterial Growth Curve
Source: https://theory.labster.com/bacterial-growth-curve/

The Bacteria that are injected into a new family do not divide right away. The time frame in which the number of cells does not rise is known as the lag phase. Bacteria are ready to divide their cells when the lag phase is over. Bacterial generation time is often measured during the log phase, when it is shortest. The growth of bacteria achieves a stationary phase when the number of cells generated equals the number of cells dying, meaning that there is no net increase in the bacterial population due to cell division and cell death. The number of bacteria continuously declines during the death or decline phase, meaning that more cells die than are generated.[**10**]

1) The simplest exponential growth model for human as well as for bacterial population due to Malthus**[11]** is $\frac{dB}{dt} = \alpha B, \quad t > 0$
Where $B(t)$ *is* the number of individual in population dynamics and $\alpha > 0$ is the specific growth rate of the population. But this model is valid in unlimited environment.
2) The population density first increases, but due to at higher density the rate of increase decreases Perl[**12**] modified the above population growth model as $\frac{dB}{dt} = \alpha B - \beta, t > 0,$
$\beta > 0$,which is the degree to which the bacterial growth reduced due to density increase.

3) Different models for growth of bacterial colonies in nutrient medium have been discussed by Kapur, Teissier, Shehata and Marr related to population growth of bacterial coloney**[13,14,15]**
4) In certain bacterial colonies, toxic substances are produced by bacteria which become a limiting factor to their further growth by rapid increase the mortality rate under toxic effects.
In the present abstract we shall develop a mathematical model for controlling the bacterial population growth which produced toxic substances under following assumption
a) Mortality depends on increasing density of population of bacteria

b) There is a rapid decrease in the number of bacteria from the environment due to production of toxic substances by bacteria and population tends to extinction.

Formulation of the model and its solution: Consider $B(t)$ the total number of bacteria at a time t. The change in the number,$\frac{dB}{dt}$ is equal to differences of birth rate and death rate.Assumeind the death rate id effected by two factors increasing density of bacterial population and secondly increasing concentration of toxic substances produced by bactria ,so the bacterial population growth model as $\frac{dB}{dt} = \alpha B - \beta B^2 - \mu c B^n, t > 0$ …(1)

Where μ is a positive constant c represents the concentration of toxic substances onwhich mortality depends.

Now we further suppose that the toxic substance being produced at a constant rate r per number of bacteria, then

$$\frac{dc}{dt} = r\,B$$

Now $$\frac{dB}{dt} = B(\alpha - \beta B - \mu c B^{n-1}) \ldots (2)$$

Or $$\int \frac{dB}{B(\alpha - \beta B - \mu c\, B^{n-1})} = t + C$$

Where C is constant of integration

The initial condition $B(0) = B_0$

Case 1: For $n = 1$, equation(2) become

$$\frac{dB}{dt} = B(\alpha - \mu c)B - \beta B^2)$$

Let the growth rate $r = \alpha - \mu c$ and carrying capacity $K = \frac{r}{\beta}(r > 0)$with $B(0) = B_0$

Then $B(t) = \frac{K}{1+\left(\frac{K}{B_0}-1\right)e^{-rt}}$ where $r = \alpha - \mu c, K = \frac{r}{\beta}$

If $r \leq 0$ then $B(t) \to 0$ as $t \to \infty$

So the bacterial population tends to zero that is population tends towards extinction.

Case 2: For $n = 2$,

$B^n = B^2$ so the mortality terms for bacterial colony becomes $(\beta + \mu c)B^2$

Now equation (2) becomes

$$\frac{dB}{dt} = \alpha B - (\beta + \mu c)B^2$$

Gives $B(t) = \frac{K}{1+\left(\frac{K}{B_0}-1\right)e^{-\alpha t}}$ where $k = \frac{\alpha}{\beta+\mu c}$

That is the growth rate of bacterial colony is α.

Results and Discussion:

Equilibria satisfy $f(B) = \alpha B - \beta B^2 - \mu c B^n = 0$

Let the Trivial equilibrium $B = 0$ and positive equilibria $B^* > 0$ satisfies

$$\alpha - \beta B^* - \mu c (B^*)^{n-1} = 0$$

For $n = 1$ give $B^* = \frac{\alpha - \mu c}{\beta}$

For $n > 1$ it isa non-linear algebraic equation existence and number of positive roots depends on parameters

Linear Stability:

Now $f'(B) = \frac{d}{dB}(\alpha B - \beta B^2 - \mu c B^n)$

$= \alpha - 2\beta B - \mu c n B^{n-1}$

(i) An equilibrium B^* is stable if $f'(B^*) < 0$ and unstable if $f'(B^*) > 0$

(ii) At $B = 0, f'(o) = \alpha$, so $B = 0$ is unstable when $\alpha > 0$ and stable when$\alpha < 0$

(iii) If $\alpha = 0$ degenerate case

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